COEFFICIENTS OF FRICTION FOR SANDSTONE SLIDING ON QUARTZ GOUGE

COËFFIÇIENTS DE FROTTEMENT POUR GLISSEMENT DE SANDSTONE SUR FRAGMENTS DE QUARTZ

REIBUNGSKOEFFIZIENTEN FÜR SANDSTEIN AUF EINEM UNZEMENTIERTEN AGGREGAT AUS QUARTZ

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SUMMARY-In triaxial sliding friction tests, precut cylinders of Tennessee Sandstone were slid on an uncemented aggregate of quartz fragments with a median diameter equivalent to that of fragments in natural quartz fault gouge (about 25μm). Below a confining pressure of 0.7 kb stick-slip was an observed mode for sliding whereas above 0.7 kb only stable sliding occurred. Coefficients of friction (μ') calculated as the ratio τ verses σ_n for each test were found to decrease with increasing confining pressure along different trends for the two modes of sliding. The coefficient of friction (μ), calculated as the slope of a curve from data of tests at a variety of normal stresses each plotted as τ verses σ_n, is also used to represent frictional characteristics of sandstone on gouge. Each mode of sliding has a unique coefficient of friction: $\mu = .72$ for stick-slip and $\mu = .68$ for stable sliding. Although the value of μ is not indicative of a particular mechanism of frictional sliding, it is sensitive to changes in the sliding mode.

ZUSAMMENFASSUNG-In Versuchen zur Gleitreibung unter drei-achsiger Belastung wurden vorgeschnittene zylindrige Proben aus Tennessee Sandstein zum Gleiten gebracht,und zwar auf einem unzementierten Aggregat aus Quartzbestandteilen mit einem mittleren Durchmesser, der dem von natürlichem Verwerfungsmaterial (gouge) aus Quartz gleicht (etwa 25μm). Unterhalb eines hydrostatischen Druckes von 0.7 kb war 'stick-slip' der beobachtete Modus des Gleitens; hingegen wurde oberhalb 0.7 kb ausschliesslich stabiles Gleiten beobachtet Reibungskoeffizienten (μ'), definiert für jeden Versuch als das Verhältnis τ zu σ_n, nahmen systematisch ab mit zunehmendem hydrostatischen Druck; jedoch geschah dies in verschiedener Weise für die zwei verschiedenen Modi des Gleitens. Wenn man den Koeffizienten μ benutzt (Berechnet als die Steigung der Kurve durch Datapunkte τ gegen σ_n von Versuchen mit verschiedenen Normalspannungen) um das Reibungsverhalten von Sandstein zu beschreiben, so ergeben sich unterschiedliche jedoch charakteristische Reibungskoeffizienten: $\mu=.60$ für 'stick-slip', und $\mu=.70$ für stabiles Gleiten. Obwohl der Betrag von μ keinen Hinweis auf einen speziellen Mechanismus des Gleitens bietet, so ist er dennoch deutlich abhängig von Modus des Gleitens.

RESUME-Pour les experiences triaxiales de glissement, les cylindres de 'Tennessee Sandstone', coupés en avance, ont été glissés sur une agregation noncementée de fragments de quartz; les derniers ayant un diametre median equivalent a cel des fragments de quartz dans une zone faillée (a pen prés 25μm). Pour des pression hydrostatique moindre de 0.7 kb, 'stick-slip' a été observé, tandis que au-desus de 0.7 kb seul glissement stable (stable sliding) a été observé. Pour tous les deux mode de glissement, on e trouvé que, le coefficient de frottment (μ') decroit guand la pression augmente. Le coefficient de frottement (μ) egale a la paute d'un graphique de τ en fonction de σ_n á été aussi utilisé pour characterise les proprietés de frottment de sandstone en presence des fragments de quartz. Les graphiques obtenus á partir des donnes aquerir pour nombrenx valeurs de pression normale montre que chaque regime de glissement a un μ characteristique: $\mu=0.72$ pour stick slip et $\mu=0.68$ pour glissement stable. Bien que μ ne soit pas indicative d'un mecanoisme particulier de glissement il (μ) est sensible aux chagements de regime de glissement.

INTRODUCTION

Geologists and engineers usually characterize frictional properties of the rock by equating the force (f_n) normal to a fracture and the shear force (f_s) parallel to that fracture necessary to initiate or maintain sliding. The simplest relationship between f_s and f_n, Amonton's Law, is that they are linearly related by a constant known as the coefficient of friction (μ) (Bowden and Tabor, 1950). Because experiments show that μ is independent of the area of contact, μ is also the ratio of shear stress (τ) across the surface to normal stress (σ_n) along the surface (Jaeger and Cook, 1969). This simple frictional law is useful because it gives a rapid means of characterising and comparing the frictional properties of various rock surfaces.

The purpose of this paper is to present data from triaxial sliding friction tests which suggest that Jaeger's (1959) interpretation of Amonton's linear frictional law is applicable when gouge is present on a sliding surface. The data, which assess the effects of quartz gouge on the frictional properties of quartzose sandstone, also demonstrate that changes in μ reflect changes in physical processes involved in frictional sliding.

A study of the effect of fault gouge on the sliding properties of rock is important because faults and shear fractures in rocks rarely consist of a single clean cut fracture; a fault breccia or gouge is often, if not always, found between the walls of a fault (Reid and others, 1913). Any attempt to characterize or model the faulting process, which may involve creep (aseismic faulting) or unstable slip (seismic faulting) must consider the influence of fault gouge.

BACKGROUND

There is some confusion in the geological literature concerning the meaning of the coefficient of friction. This is due in part to the complicated nature of sliding friction and in part to the lack of a standardized method for reducing experimental data to obtain the coefficient of friction.

The physical process of frictional sliding is influenced by many interrelated properties of the rock. Rock surfaces are not smooth; movement occurs when brittle asperities either climb over each other, plough through others, or shear off (Jaeger and Cook, 1969). The asperity may also deform without shearing off, if the yield stress of the asperity is exceeded (Logan and others, 1973). Because these frictional phenomena are so interrelated any simple law of friction does not model the physical processes of frictional sliding. Thus, it is not clear that the coefficient of friction is a sensitive measure of the frictional properties of a rock surface.

In the literature there is more than one method for evaluating the frictional properties of rock by calculating a coefficient of friction. In some cases a coefficient of friction (μ') is determined for each experiment where μ' is the ratio τ/σ_n as calculated from the data at some point along the force-displacement record of that experiment. μ' is found to decrease with increasing normal stress for sandstones and dolomites (Handin, 1969; Mauer, 1965). Logan and others (1973) find that μ' for limestones is not a function of normal stress. When μ' is used to represent the frictional properties of rocks, Amonton's Law is valid only if the value of μ' is independent of normal stress. However, another method of reducing frictional data gives a coefficient of friction (μ) which is generally independent of normal stress. Jaeger (1959), Hoskins and others (1968), and Jaeger and Rosengren (1969) calculate a coefficient of friction (μ) for fracture surfaces by reducing data from several experiments among which the normal stress is varied. In this case τ is plotted against σ_n and μ is the constant slope of the curve which fits that data points. Often this relation between τ and σ_n contains a second constant which is the equivalent of the cohesion term in the Mohr-Coulomb fracture criterion (Jaeger and Cook, 1969). The cohesive shear term (τ_o) represents the τ-intercept of Jaeger and Cook's friction curve:

$$\tau = \tau_o + \mu\sigma_n$$

This is a modified form of Amonton's Law. In other instances the experimentally determined relation between τ and σ_n is nonlinear; Mauer (1965) and Murrell (1965) indicate that their data for sliding friction tests fits a power law between τ and σ_n:

$$\tau = \mu\sigma_n^X$$

where μ is a constant and independent of normal stress across the sliding surface. Dieterich (1971) presents some data which fits neither a power law or the linear law.

To date no information is available on the relation among experimental results from different testing apparatuses. Frictional testing apparatuses in use include: two-block direct shear apparatus (Coulson, 1970; Patton, 1966; Pratt and others, 1972), three-block direct shear apparatus (Dieterich, 1972; Hoskins and others, 1968; Jaeger and Rosengren, 1969; Mauer, 1965), triaxial apparatus (Brace and Byerlee, 1966, Byerlee, 1967; Handin, 1969; Logan and others, 1973; Murrell, 1965; Raleigh and Paterson, 1965) and biaxial shear apparatus (Scholz and others, 1972).

An exact meaning of the coefficient of friction is further complicated because geologists have not picked data at a consistent point in a specimen's displacement history. Thus there is often a failure to distinguish among a coefficient of friction at the initiation of sliding (μ_i or μ_i') (Handin, 1969; Logan and others, 1973), a maximum coefficient of friction (Byerlee, 1967), a residual or kinetic coefficient (μ_k or μ_k') (Hoskins and others, 1968; Mauer, 1965; Murrell, 1965; Raleigh and Paterson, 1965) and a static coefficient of friction calculated at the onset of each slip event during stick-slip (μ_s') (Dieterich, 1972; and Scholz and others, 1972). Although the coefficient calculated at the initiation of a sliding friction experiment is a static coefficient, it is calculated for a surface with little surface damage due to sliding whereas the static coefficient calculated at the initiation of slip during stick-slip represents the frictional properties of a surface in which a certain amount of gouge is generated. This difference is equivalent to either determining the coefficient for an extension fracture with no gouge or to determining the coefficient for a shear fracture which already exhibits gouge and has a history of sliding. A distinction is also necessary between the kinetic coefficient for an experiment in which sliding is stable and that during which sliding is unstable. For stick-slip the assumption is made that the seismic efficiency is zero and μ_k is calculated at half the height of the force drop during unstable slip. μ_k for stable sliding is calculated from a smooth force curve during uninterrupted slip.

The τ/σ_n ratio may vary in one of several ways during a friction test. Jaeger and Rosengren (1969) show that the ratio continuously increases during the experiments with a three block direct shear apparatus. Byerlee (1967) shows that the ratio varies in two different ways in triaxial tests. From an initially low point the ratio increases to a maximum and then decreases. In other experiments Byerlee (1967) show that the ratio continuously decreases from an initial maximum. Coulson (1970) has found four different variations of μ_k' with displacement during a two block direct shear test. The maximum coefficient of friction and μ_k' are the same only if μ_k' continues to increase with displacement through the duration of the experiment. However, the maximum resistance to sliding may occur very early in the experiment and therefore, is higher than the residual resistance. From these tests it is clear that μ_k' is a function of the position from where the data are picked during the displacement history. The point from where the ratio is picked should be determined by the nature of the experiment and the type of information required from the experiment.

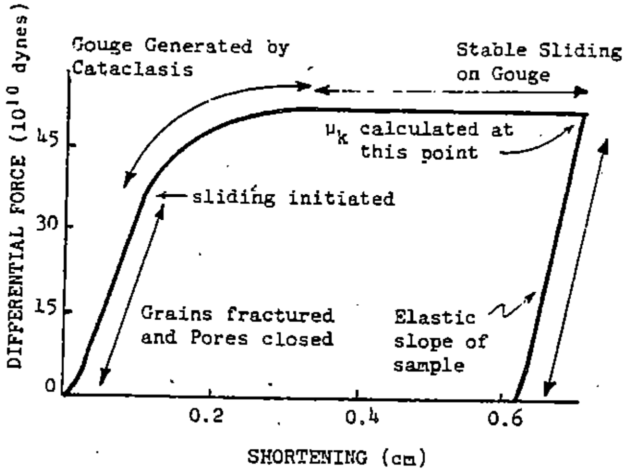

Figure 1. Differential force-shortening curve for precut Tennessee Sandstone sliding stably on a 0.12-cm thick layer of artificial, dry quartz gouge. Experimental conditions include a confining pressure of 1.0 kb, displacement rate of 10^{-3} cm/sec, and room temperature.

Thus the basic problems exist with the present non-uniform methods of reporting frictional data. It is necessary to distinguish μ and μ' as well as establish a consistent point in the displacement of the sample for calculating μ and μ'.

EXPERIMENTAL PROCEDURE

Precut cylinders of Tennessee Sandstone were used to obtain data on the effect of dry quartz gouge on the mode of sliding and the kinetic coefficients of friction at confining pressures to 2.7 kb and a displacement rate of 10^{-3} cm/sec. The cylindrical specimens, which are 5 cm in diameter and 10-cm long, were precut at approximately 35° to the cylinder axis and ground to ± 0.1° with an 80-grit wheel. The ends of the specimens were ground parallel to within 0.001 cm. Quartz grains between 100 and 250-μm in diameter were evenly distributed in a 0.20-cm layer along the precut. These quartz grains were cataclastically reduced during sliding to form an artificial quartz gouge with a uniform thickness. A 0.6-cm thick steel spacer powdered with MoS_2 on both sides was placed between the specimen and the upper piston in order to decrease friction between them and thus improve reproducibility. The 5 by 10-cm specimens were then jacketed by heat-shrink, polyolifin sleeves.

In addition, cylinders of intact, dry Coconino Sandstone were fractured and shortened different amounts by sliding along the induced fractures to simulate the natural development of fault gouge from host rock. These tests were done at confining pressures to 0.5 kb and a displacement rate of 10^{-3} cm/sec, and they gave data on the coefficient of friction for a fracture surface containing gouge.

A typical force-shortening curve is shown in Figure 1 for precut Tennessee Sandstone stable sliding on an 0.12-cm thick layer of dry gouge which is generated from a 0.2-cm thick layer of sand. A thin section study from tests at increasingly larger displacements shows the following correlation between the generation of quartz fault gouge and the force-shortening curve. Compaction and some cataclastic deformation of the sand occurs during the initial application of hydrostatic pressure on the specimen. These continue during the initial increase of differential stress. Because of this inelastic deformation, the slope of the initial part of the loading curve is nonlinear and not as steep as that of the linear unloading curve. The host rock behaves essentially as an elastic body throughout the experiment. At the "yield stress" sliding on the layer of sand begins and it is accompanied by still more intense cataclastic deformation. The grain size of the quartz sand between the precut samples decreases as displacement progresses. As cataclasis continues, the load necessary to maintain sliding increases until the ultimate strength is reached. Once the ultimate strength is reached, the resistance to sliding does not vary appreciably. At this point the size distribution of the experimental quartz aggregate approaches that of natural quartz gouge found in both faults and fractures in sandstone and also formed by cataclasis (Engelder, 1973). For this reason the experimental quartz aggregate is referred to as quartz gouge. The steady-state portion of the force-shortening curve is corrected for offset of the specimen halves and accompanying decrease of area of contact. Coefficients of friction are calculated from force data at 0.6-cm of sample shortening by making an appropriate adjustment for a changing sliding surface contact area.

EXPERIMENTAL RESULTS

The confining pressure effected the sliding mode of Tennessee Sandstone on quartz gouge (Figure 2). For all experiments stable sliding occurred during the initial cataclastic deformation of the original layer of sand. This was followed by stick-slip when the confining pressure is less than 0.7 kb. Stick-slip did not occur within the limits of the experimental displacements if the confining pressure was above 0.7 kb. The onset of stick-slip occurred earlier in the displacement history of the specimen as the confining pressure increased from 0.14 kb to 0.5 kb. One might expect stick-slip to occur with even less displacement at 0.7 kb confining pressure, but this was not what happened. At 0.7 kb the sliding was stable at displacements greater than those for which stick-slip occurred at 0.14 kb. Thus there was a stick-slip to stable-sliding transition as a function of confining pressure. Apparently a fundamental change in the sliding process occurs at this transition.

For all experiments at less than 0.7-kb confining pressure the gouge at the contact with Tennessee Sandstone was non-indurated (regular) and had a median grain size of 25μm. For experiments at confining pressures above 0.7 kb, gouge at the contact with the Tennessee Sandstone was indurated and had a median grain size of 0.5 μm (Engelder and McKee, 1973). The change in gouge formed at the contact corresponds with the change in mode of sliding. The nature of the gouge at the Tennessee Sandstone contact apparently influences the frictional properties of the Tennessee Sandstone but the reason for the change in sliding mode is not clear.

The change in sliding mode also correlates with a change in the coefficient of friction of Tennessee Sandstone sliding on quartz gouge. A kinetic

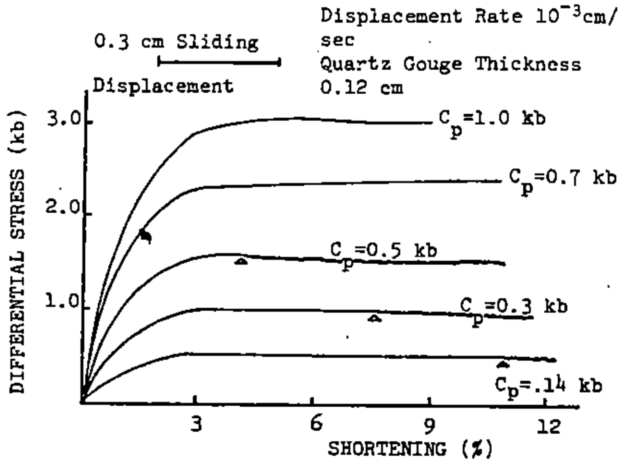

Figure 2. Differential stress-shortening curves showing the effect of confining pressure on frictional sliding of precut specimens of Tennessee Sandstone with a compacted 0.12-cm thick layer of dry quartz gouge. Smooth lines indicate stable sliding and jagged lines indicate stick-slip. Triangle marks the onset of stick-slip.

coefficient of friction (μ_k') is calculated for each experiment from force data at 0.8 cm of sliding

$$\mu_k' = \tau/\sigma_n.$$

For experiments which slid by stick-slip μ_k' was calculated from the mean value of the differential stress; that is at a point midway between the maximum and minimum stress of the stick-slip cycle. Changes in μ_k with respect to normal stress across the precut surfaces of Tennessee Sandstone sliding on quartz gouge are systematic and reflect the change in sliding mode (Figure 3). The value of μ_k' decreases from 0.88 as the normal stress increases for the stick-slip mode of sliding. At a normal stress of about 1.4 kb, the change in mode from stick-slip to stable sliding is accompanied by an increase in μ_k from 0.68 to 0.72. At the lower normal stress μ_k' decreases systematically whereas μ_k' approaches a constant value along a different trend at high normal stresses. Separate lines are drawn through μ_k' for stick-slip and for stable sliding to suggest that the data belong to different trends.

In order to determine the experimental reproducibility and thus the range of μ_k' among several experiments, up to five similar specimens are deformed. At 1.0 kb confining pressure μ_k' falls between 0.69 and 0.72 and has a standard deviation of 0.01. This standard deviation is assumed to apply to all other experiments because μ_k' calculated for two or more repeated tests never varies by more than 0.02 except at very low normal stresses.

In order to discover if Amonton's Law applies to Tennessee Sandstone sliding on quartz gouge, the data are plotted with shear stress as a function of normal stress (Figure 4). This plot shows two linear trends which differ in slope and τ-intercept. The lines represent a modification of Amonton's second law (Jaeger and Cook, 1969)

$$\tau = \tau_o + \mu_k\sigma_n$$

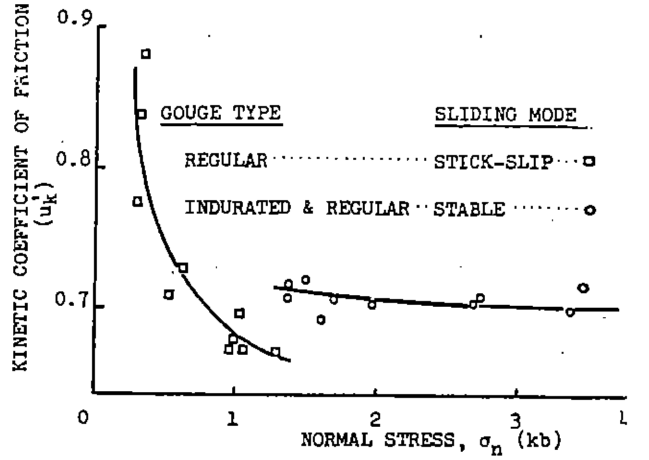

Figure 3. Kinetic coefficient of friction (μ_k') vers normal stress for precut specimens of Tennessee Sand stone sliding on a compacted quartz gouge.

where the constant μ_k is the slope of the line and τ is the τ-intercept. According to Amonton's Law μ_k is the kinetic coefficient of friction of the slidin surface over a particular experimental range of σ_n. In this situation the two sliding modes are represen ed by a unique μ_k which is not a function of normal stress. At the lower normal stresses where stick-sl is observed, μ_k is 0.60 and τ_o is 80 bars; at higher normal stresses where stable sliding is observed, th μ_k is 0.70 and τ_o is 50 bars. The reduction of values for force from any point along the stick-slip loading cycle results in τ verses σ_n data points wit a curve whose slope differs by 4° from the slope derived from the stable sliding data.

For precut Coconino Sandstone sliding on quartz goug the μ_k and τ_o are the same as those for Tennessee Sandstone (Engelder, 1973). This similarity is important in light of the fact that the fracture streng of the Coconino is 20 to 30% less than that of the Tennessee Sandstone. This suggests that the mechanical properties of the quartz fault gouge influence the frictional sliding of a fracture more than do the mechanical properties of the host sandstone within which the gouge forms.

Solid cylinders of Coconino Sandstone were fractured and then slid in order to obtain a μ_k and τ_o for a fractured surface separated by gouge. These experiments test the effect of an undulatory fracture surface on the μ_k and τ_o for a sandstone on gouge. At normal stresses to at least 1.0 kb a layer of gouge nearly 0.15-cm thick is generated. In this case bot μ_k (0.72) and τ_o (120 bars) are higher than they are for Tennessee Sandstone with precut surfaces (Figure 4).

DISCUSSION

Jaeger and Cook (1969) point out that the plot of μ_k' verses σ_n is more complicated that a plot of μ_k vers σ_n. In the former case the coefficient of friction varies as a function of confining pressure whereas in the latter case the coefficient is independent of confining pressure for some range of σ_n. The primar advantages of the latter with respect to sliding on quartz fault gouge are that a change in sliding mode

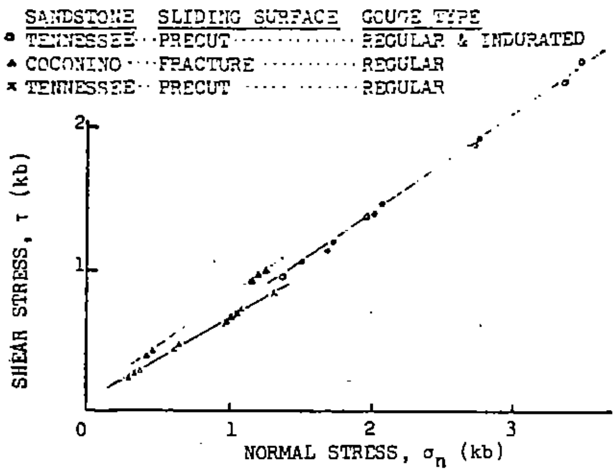

Figure 4. Shear stress versus normal stress data for sliding of precut Tennessee Sandstone on a 0.12-cm thick layer of dry quartz gouge and fractured Coconino Sandstone on a 0.10 to 0.15-cm thick layer of quartz gouge.

is characterized by a change in μ_k but for each mode μ_k is unique. Only two constants, τ_o and μ_k are necessary to represent the frictional resistance of the sliding surface. Both of these constants are sensitive to changes in frictional characteristics of the sliding surface without being effected by increasing normal stresses. In contrast there are trends in μ_k' for each sliding mode but there is no unique value for μ_k' and in some instances μ_k' has the same value for both modes of sliding (Figure 3).

The difference between μ_k' and μ_k is that in the latter case the term τ_o has been subtracted from the shear stress necessary to maintain sliding:

$$\mu_k = \frac{\tau - \tau_o}{\sigma_n}$$

τ_o is a constant which when added to the numerator gives the μ_k' increasingly larger values as τ and σ_n decrease. But if the rock surface does not have a characteristic τ_o, then $\mu_k = \mu_k'$.

The frictional data for sandstone on quartz gouge plotted in τ-σ_n space has a τ-intercept which Jaeger and Cook (1969) suggest is due to a fundamental cohesive shear strength of the surfaces. The quartz gouge experiments show that τ_o exists even when surfaces are sliding stably and all major asperities have been sheared from the sliding surface. Likewise, the curves in Figure 4 indicate that the sliding surface is represented by a unique τ_o for each sliding mode. It may be that the 'cohesive' strength between sliding surfaces represents the total strength of microscopic interlocked asperities.

If τ_o represents the fundamental shear strength of the sliding surface, a change in size or strength of asperities on the sliding surface should effect τ_o. For sliding on quartz gouge it was found that asperities (1 cm^2 in area with 0.1 cm of relief) due to the uneven fracture surface of Coconino Sandstone result in an increase of both τ_o and μ_k compared with those for precut Coconino Sandstone. Thus the term τ_o is sensitive to changes in shear strength of the sliding surface. The increase in μ_k due to asperities may be caused by increased frictional resistance of asperities with surfaces inclined to the sliding surface. This suggestion is based on Patton's (1966) observation that inclined teeth on the surface of plaster cause a change in slope of the maximum shearing strength curve in τ-σ_n space.

μ_k rather than μ_i and μ_s is used to represent the frictional properties of sandstone sliding on quartz fault gouge because the presence of gouge indicates a displacement history and μ_k represents the sliding resistance of gouge to sandstone in motion. Once an 0.12-cm thick layer of gouge has formed on the sliding surface the value of μ_i or μ_s and μ_k differs by less than 2% and μ_k is equal to μ_s when stable sliding occurs (Engelder, 1973). Although the use of μ_k and its similarity to μ_i is applicable to sandstone on quartz gouge, there is a greater difference between the values of μ_i, μ_k, and μ_s for stick-slip on clean surfaces of granite (Byerlee, 1967). In this latter situation μ_s may be more meaningful in characterizing the frictional properties of the rock involved in the experiments.

CONCLUSIONS

The purpose of this paper is to emphasize the need for a uniform system for measuring and reporting the frictional characteristics of rocks. Based on the examples presented in this paper, it seems that frictional data reduced to a plot in τ-σ_n space are most versatile. In this case Amonton's Law is satisfied and the frictional properties of the sliding surface may be represented by the constants μ_k and τ_o. However, when frictional data is reduced it should be made clear where from individual force-displacement curves τ and σ_n are picked.

Indurated gouge on the sliding surface causes a fundamental change in frictional properties of the sandstone; μ_k increases when the indurated gouge is present. Although μ_k does not indicate the mechanism of frictional sliding, it is a sensitive indicator of change in the sliding process.

ACKNOWLEDGEMENTS

This paper is based on a dissertation submitted in partial fulfillment of the requirements for a Ph.D. in Geology at Texas A&M University. The use of facilities at the Center for Tectonophysics, Texas A&M University, and the advice of Professor John Handin and his associates are greatly appreciated. The work was supported by the U.S. Geological Survey and the Advanced Research Projects Agency Order No. 1684.

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