Reprinted from Mechanical Behavior of Crustal Rocks
Geophysical Monograph 24

Fluid Flow Along Very Smooth Joints at Effective Pressures up to 200 Megapascals

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Flow rates and thus permeability were measured within very smooth joints in Cheshire quartzite. Measurement of the change in aperture with effective pressure shows that at effective pressures of less than 20 MPa, changes in confining pressure have a larger influence on the permeability than changes in pore pressure. Although a 'cubic law' model for flow within a joint gives a rough estimate of joint permeability, the use of joint aperture as the only variable in the cubic law model for flow is inadequate for calculating joint permeability. We suggest that the effective cross section available for flow changes with effective pressure in a nonlinear manner and we present a modified cubic law to account for this behavior.

Introduction

Flow of fluid in rocks of very low permeability is primarily along joints. At midcrustal depths the degree to which joints close under pressure ultimately influences the bulk permeability of rocks with low intrinsic permeability. Here we examine some details concerning the effect of joint closure on joint permeability.

The rate of flow of a fluid along a nominally closed joint is related to the aperture of the joint. For parallel plates there is a direct relationship between aperture and fluid flow based on the Hele-Shaw model [*Harr*, 1962]. This relationship can be developed from Darcy's law and simplified as

$$Q/\Delta h = Cd^3 \qquad (1)$$

where Q is the flow rate, Δh is the drop in head, d is the joint aperture, and C is a function of fluid properties as well as geometry of the flow path [*Witherspoon et al.*, 1980]. We shall refer to this relationship as the cubic law, and its validity has been discussed in a series of papers including those by *Lomize* [1951], *Sharp* [1970], *Sharp and Maini* [1972], and *Gale* [1975]. *Gangi* [1978], using the experimental data of *Nelson* [1975], argues that while the permeability of a joint may be stress history dependent, it is uniquely defined by the joint aperture. The stress history determines the asperity distribution function of the joint and, consequently, its aperture for a given stress. The experimental data of *Witherspoon et al.* [1980] confirm the validity of the cubic law for laminar flow along tensile joints with apertures down to 4 μm and effective pressures of 20 MPa. They also argue that permeability is uniquely defined by joint aperture. *Kranze et al.* [1979] present experimental data on very smooth joints which indicate that parameters must be added to the simple cubic law to account for stress history. The purpose of this paper is to present data suggesting that for very smooth joints the rate of change of a joint's effective cross section with pressure is higher than the rate of change of a joint's closure with pressure. Hence one additional parameter accounting for effective cross section must be included in the cubic law to make it a more general equation. By effective cross section we mean the area in an imaginary plane intersecting a joint through which fluid can flow unobstructed by asperities.

Experimental Procedure

Our experimental procedure for fluid flow tests was much the same as described by *Kranz et al.* [1979]. Experiments were conducted in a triaxial, servocontrolled, hydraulic press equipped with a 5.08-cm bore pressure vessel. Kerosene was used as the pressure medium as well as the fluid pumped through the rock. It is chemically inert with respect to the rock, so that only mechanical effects of pressure were investigated. The confining pressure system was independently controlled and separate from the internal fluid system. All samples were covered with a polyolefin jacket. Steel end caps with center holes and radial grooves were affixed to the samples.

Split cylinders were used to model joints. A block of Cheshire quartzite was saw cut into large prismatic sections with ground sides. Two sections with parallel sides were clamped together and cores, also approximately 3.5 cm in diameter, were taken centered on the joint between the sections. These split, cylindrical samples were then

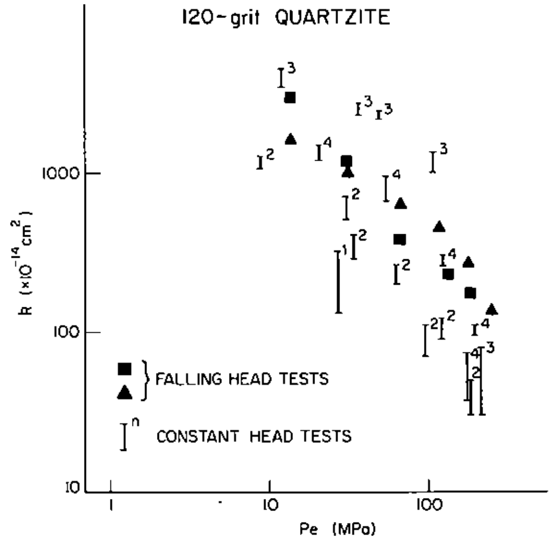

Fig. 1. Whole rock plus joint permeability k at various effective pressures P_e for joints in Cheshire quartzite prepared with 120-grit polishing compound. Constant head data come from four different samples (1-4), whereas falling head data come from two different samples. The error bar for permeability shown for constant head tests indicates the variability of k for repeated tests during which confining pressure is held constant and hydraulic head is varied.

reclamped and saw cut to be approximately 9 cm in length, and the ends were ground parallel. The split samples were then unclamped and the interior, opposing surfaces were ground with either number 80 grit or 120 grit.

Changes in joint aperture were measured with a four-armed dilatometer, consisting of four thin beryllium-copper beams connected in a radially symmetric pattern to an aluminum ring which was slipped over the sample. The output from two arms measuring rock compression was subtracted from the output of the other two arms measuring the sum of joint closure and rock compression so that the combined output was proportional to joint closure. Calibration included subtracting the effect of pressure on the dilatometer. Measurements made with the dilatometer do not give an absolute value for the joint aperture but rather a change in aperture. To get an absolute value, the joint must be closed down to the point where no further changes in aperture are discernable, and that point taken as $d = 0$. We did not reach such a point in any of our experiments. From the asymptotic approach to complete closure we can, with some uncertainty, get absolute values of d as a function of $P_c - P_f$. At low pressures the estimated uncertainty in the aperture may be as high as 200%, but it decreases rapidly with pressure, so that above 10 MPa the estimated uncertainty in the aperture of the joint may be considered as ±25%.

For the experiments reported here, the fluid flow apparatus was modified to measure flow rates during constant head tests, whereas *Kranz et al.* [1979] reported data from falling head tests. For the constant head test the pore fluid systems at the top and bottom of the sample were isolated. The pore pressure on the high-pressure end of the sample was generated by servocontrolling a double-acting Aminco pump. Feedback for servocontrolling the pump was from a BLH pressure cell. Pore pressure on the low-pressure end of the sample was maintained with a servocontrolled piston moving within a small pressure vessel serving as a pore fluid collector. Here again the feedback element was another BLH pressure cell. The signals from the two BLH pressure cells were compared to maintain a constant pore-pressure gradient with the signal driving the Aminco pump as the master maintaining a constant absolute pore pressure. The BLH pressure cells were capable of detecting pressure differences of 0.01 MPa at 200 MPa ambient pressures. Using a linear variable displacement transducer (LVDT), flow rate was measured by monitoring the withdrawal of the piston from the pressure vessel collecting pore fluid. Strokes of the Aminco pump were not used to measure flow because of back leaks through ball-check valves.

Pulse Decay Versus Constant Head Tests

The purpose in using a constant head test was to check data using the falling head tests, as reported by *Kranz et al.* [1979]. The constant head test assumes that Darcy's law holds and there is a linear pressure gradient along the length of the sample. That is,

$$Q = \frac{kA}{\mu}\frac{dP}{dL} \tag{2}$$

where Q is the volume flow rate, k is the intrinsic permeability, A is the cross section through which the fluid flows, μ is the dynamic viscosity, L is the length of flow path, and P is the fluid pressure. For the constant head test, Q is measured as the rate of flow into the small pres-

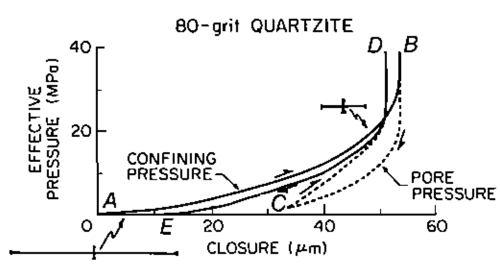

Fig. 2. Joint aperture versus effective pressure where the solid curve represents closure during adjustment of confining pressure and the dashed curve represents closure during adjustment of pore pressure. The joint in Cheshire quartzite was prepared with 80-grit polishing compound. By the nature of the experiment, confining pressure is applied before pore pressure and removed after pore pressure. Pore P_p and confining P_c pressure conditions are as follows: point A, $P_c = P_p = 0.1$ MPa; point B, $P_c = 40$ MPa and $P_p = 0.1$ MPa; point C, $P_c = 40$ MPa and $P_p = 39.4$ MPa; point D, $P_c = 40$ MPa and $P_p = 0.1$ MPa; and point E, $P_c = P_p = 0.1$ MPa.

sure vessel from the sample. Permeability may be calculated using either the cross section of the sample (9.62 cm^2) or the cross section of the joint whose aperture is estimated using the four-armed dilatometer. If the cross section of the sample is used for A, k is then 'rock permeability' in which the major flow capacity is governed by the joints. This is to be distinguished from 'joint permeability' which is calculated using the effective cross section of the joint. The ratio of rock permeability for rocks in which the major flow capacity is contributed by joints to joint permeability is equal to rock porosity due to joints [*Jones*, 1975].

The procedure for both the constant head and falling head tests was to prepare a sample for tests at a variety of effective pressures. For data shown in Figure 1 the tests started at low effective pressures with changes in effective pressure accomplished by increasing the confining pressure. For the falling head tests, one pulse-decay curve was recorded at each effective pressure, whereas several flow tests were recorded for each effective pressure during the constant head tests. In the latter case each flow test was accomplished at a different pore pressure gradient with a maximum range from 0.6 to 0.006 MPa/cm. Error bars in Figure 1 represent the range of permeabilities measured using a range of pore pressure gradients. The data in Figure 1 represent 220 flow tests.

Constant head tests show the same range in permeability as falling head tests for a joint prepared with 120 grit in Cheshire quartzite (Figure 1) where rock permeabilities are calculated. For comparison, intact Cheshire quartzite has a permeability of less than 10^{-14} cm^2 at effective pressures of less than 10 MPa. The variability in permeability at any effective pressure is in part associated with the preparation of six different samples for which data are shown. The grinding of surfaces by hand rarely produces a uniform surface finish, and so the initial surface finishes were probably different from sample to sample. The data converge at high effective pressures, suggesting that initial sample mismatches contribute larger variabil:ty to the permeability than would be the case if each sample assembly were exactly like its prede-

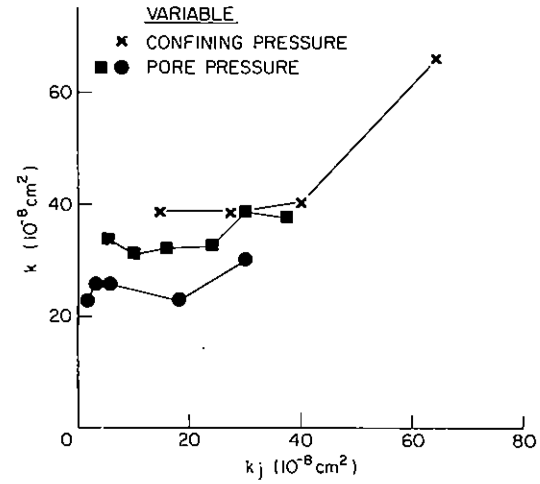

Fig. 4. Plot of joint permeability using the cubic law for fluid flow versus joint permeability using experimental data. Effective pressure was changed by changing either pore pressure or confining pressure with the other held constant.

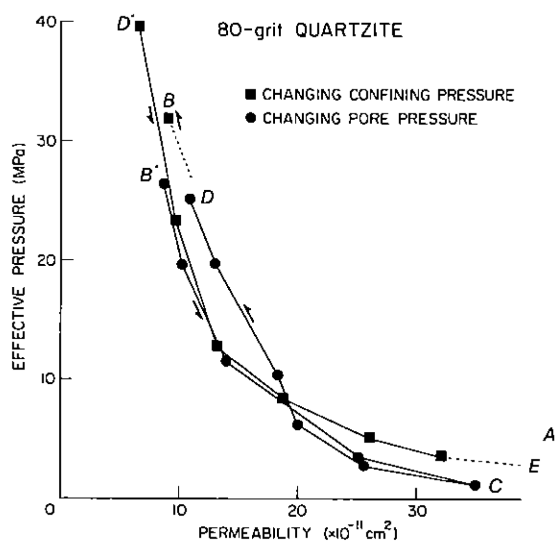

Fig. 3. Effective pressure versus permeability curves for a joint in Cheshire quartzite prepared with 80-grit polishing compound. Data come from constant head flow tests. Arrows indicate direction of change of either confining or pore pressure. Only one point was taken for permeability during initial application of confining pressure.

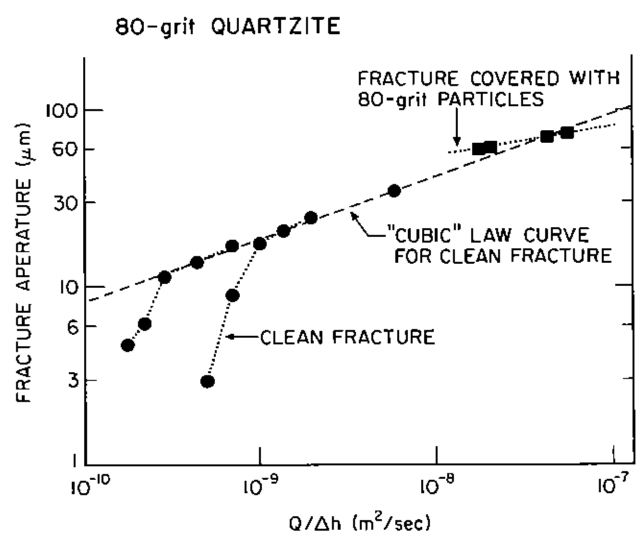

Fig. 5. Fluid flow versus aperature for clean joints and a joint lined with a layer of 80-grit particles. For the covered joint $Q/\Delta h$ decreases faster with d than as d^3, whereas for the clean joint at high pressure the opposite is true.

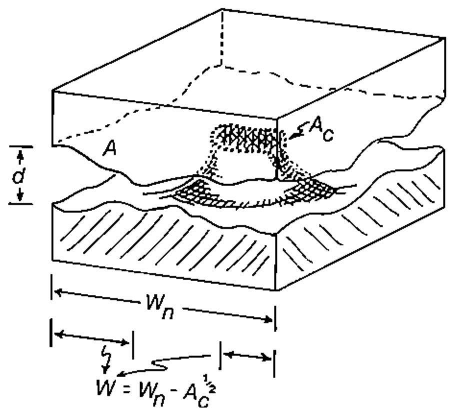

Fig. 6. Schematic of a joint where A is the nominal surface area, A_c is the real area of contact, d is mean aperture, W_n is nominal width of joint, and W is the effective cross-sectional width.

cessor. During constant head tests at a single effective confining pressure the use of different hydraulic heads gives different values for permeability. This may be attributed to different gradients in effective pressure down the length of the sample. Further variability may also arise from a slight misalignment of the saw cut sample and shear stresses at the piston-sample interface, causing part of the saw cut to pop open and thus have an excessively large aperture. From the comparison of constant head and falling head tests we conclude that both types of tests showed similar flow rates and flow rates varied in a similar manner with similar changes in effective pressure.

Change in Aperture With Pressure

Permeability of soil, intact rocks, and jointed rock follows a hysteresis loop when either confining pressure or pore pressure is cycled up and down [*Freeze and Cherry*, 1979; *Kranz et al.*, 1979; *Witherspoon et al.*, 1980]. Direct measurement of the aperture in jointed rock shows that the hysteresis in permeability is a direct consequence of the hysteresis in the response of the aperture to changing pore pressure and confining pressure [*Kranz et al.*, 1979; *Witherspoon et al.*, 1980]. The flow rates along joints are controlled by the cross-sectional area of the flow channel.

The effect of changing confining and pore pressure on aperture is shown in Figure 2. This curve is representative of four samples prepared with 80-grit polishing compound. Eighty grit was used to give joint surfaces capable of closing 40–60 μm. We found that the 15-μm total closure for 120-grit surfaces was too small to measure accurately aperture changes accompanying small changes in effective pressure. The error bars shown in Figure 2 represent the variation in position of the curves for the four experiments. Variation in position does not affect the shape or relative positions of the confining and pore pressure curves for each test. The loading path consisted of increasing the confining pressure to 40 MPa (from A to B) then increasing pore pressure to about 39.4 MPa (B to C). Pore pressure was then decreased to 0.1 MPa (C to D) followed by the release of confining pressure (D to E). Hysteresis loops are evident for both pore and confining pressure cycles, and the top of the confining pressure cycle is offset by the residual amount of the pore pressure hysteresis loop. Likewise, the bottom of the confining pressure cycle does not return to 0 μm. During the pore pressure cycle, work was lost. This behavior is displayed in the conventional sense for the confining pressure cycle where lost work is represented by the area between the loading and unloading curves.

Kranz et al. [1979] present permeability as a function of pressure with the equation

$$dk = -a\, dP_c + b\, dP_f \tag{3}$$

where $b/a < 1$ for jointed Barre granite. The physical reason for $b/a < 1$ is apparent from Figure 2. At effective pressures of less than 20 MPa, changes in confining pressure have a greater effect on changes in aperture than equivalent changes in pore pressure. A confining pressure of 40 MPa closed the joint in quartzite about 50 μm, whereas the injection of about the same pore pressure reopened the joint a little more than 20 μm. During the first cycle, such behavior may be attributed to changes in the asperity height distribution which changes less dramatically in the second cycle. However, for the second cycle in closing and opening the joint the change of confining pressures still has a larger effect despite the reverse order in which the changes took place. This result is unusual in light of several studies showing that permeability is governed by effective confining pressure and not greatly dependent on the absolute values of either pore pressure or confining pressure [*Jones*, 1975].

Both the confining and pore pressure curves exhibit marked changes in slope when the effective pressure is less than 25 MPa. However, close inspection shows that the change in slope for the confining pressure curve occurs

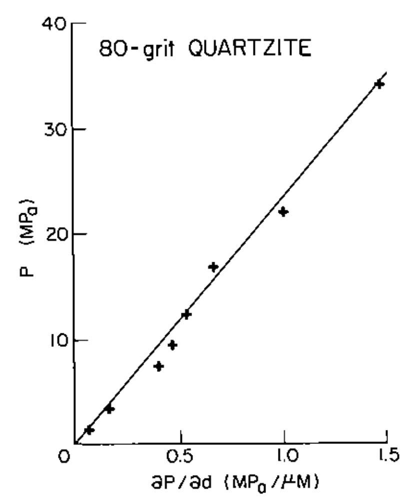

Fig. 7. Plot of joint stiffness $\partial P/\partial d$ versus confining pressure P. Data from loading curve in Figure 2.

at about 23-24 MPa, whereas the change in slope for the pore pressure curve is somewhat lower (~21 MPa). Again, this demonstrates the greater overall capacity for changes in confining pressure to affect joint aperture.

Joint Permeability Versus Effective Pressure

Change in flow rate was measured in conjunction with change in joint aperture for the split samples of Cheshire quartzite prepared with 80 grit. To illustrate details associated with change in effective pressure, the rock permeability was calculated using the cross-sectional area of the sample (9.62 cm^2). Figure 3 shows that the permeability versus effective pressure curves are similar to the joint aperture curves for the same experiment. The letters A through E correspond to the same steps shown in Figure 2. Although the absolute value of the permeability was different for each of four experiments, the relative shape and position of the curves for each experiment are consistent. During the application of confining pressure (A to B) only one permeability test was attempted for the experiment shown in Figure 3. However, consider the pore pressure cycle with confining pressure held constant (dots) during which the effective pressure was first decreased (B to C) and then increased in a series of steps (C to D). The permeability was lower during the initial decrease in effective pressure than upon the return to the same effective pressure. This is equivalent to the slight opening of the joint aperture seen during the pore pressure cycle in Figure 2. Likewise, during the confining pressure cycle with constant pore pressure (squares), permeability should have been higher during confining pressure changes that lowered the effective pressure (D to E) relative to the pore pressure cycle (C to D). This effect actually appears at effective pressures of less than 10 MPa where there is a distinct difference in the closure curves D to E and C to D (Figure 2). At low effective pressures, permeability changed more rapidly with changing confining pressure (Figure 3). At low confining pressures the permeability was higher than for high confining pressures at the same effective stress. Hence permeability varies in a complicated manner with effective pressure and is highly dependent on stress history.

Joint permeability may be calculated using the effective cross-sectional area of the joint, which is joint aperture times the diameter of the sample. Assuming that the effective cross-sectional area of the joint changes linearly with the joint aperture, this calculation requires the absolute aperture which can only be estimated [*Witherspoon et al.*, 1980]. For our calculation we assume that at the lowest effective stress, flow follows the cubic law where $k_j = d^2/12$. Knowing the flow rate and drop in head for permeability tests at low effective stress, we find the aperture d_0 which sets the k_j equal to $k = (\mu Q/A)(dL/dP)$. Once d_0 is found, we calculate k for higher effective pressures using an aperture $d = d_0 - \Delta d$, where the change in aperture Δd is measured experimentally. A plot of k_j versus k shows that measured fluid flow does not follow the d^3 law at high pressure (Figure 4). At low effective pressures and wide apertures the d^3 law is confirmed, as shown by the one-to-one change of k versus k_j for the confining pressure curve. However, for the two pore pressure curves with narrow apertures there is just a slight increase in joint permeability k with increase in aperture versus an apparent dramatic increase in k_j.

Failure of the plot of k versus k_j to follow a straight line suggests that the cubic law for fluid flow requires an additional parameter other than joint aperture. This is also seen in a plot of flow rate per hydraulic head Q/h versus aperture d (Figure 5). At high flow rates and wider apertures the curves appear to parallel a d^3 law, but at low flow rates and narrower apertures the d^3 law is inadequate.

Discussion

Our significant results include confirmation of the d^3 law at low pressure for clean joints, as did *Witherspoon et al.* [1980]. However, for filled joints permeability decreases faster with d than as d^3 and for clean joints at high pressure the opposite is true. We also reaffirmed the result of *Kranz et al.* [1979] that pore pressure has a weaker effect on fracture closure (permeability) than confining pressure.

One can begin to understand these results by considering the theory of closure of random surfaces of *Greenwood and Williamson* [1966] which has recently been partially confirmed for rock by *Walsh and Grosenbaugh* [1979]. This theory has the result that if two surfaces of random topography are in contact under a pressure P and if the deformation is elastic, the mean aperture d will be

$$d = d_0 - \Lambda \log (P/P_0) \tag{4}$$

where Λ is the standard deviation of the topography of the surface and d_0 the aperture at P_0, a reference pressure. An additional result, originally from *Archard* [1957], is that the real area of contact, A_c, is dependent only on the load and not on the nominal pressure P; thus

$$A_c = \epsilon PA \tag{5}$$

where ϵ is an elastic constant. We equate W, the effective cross-sectional width available for flow, and the real area of contact:

$$W = W_n - (\epsilon PA)^{1/2} \tag{6}$$

where W_n is the nominal width, and from (4),

$$W = W_n - \left[w_0 + (\epsilon A P_0)^{1/2} \exp \left(\frac{d_0 - d}{2\Lambda} \right) \right] \tag{7}$$

where w_0 is the contact cross section at zero pressure (Figure 6). Substituting this into equations (2) and (4) of *Witherspoon et al.* [1980] for linear flow through a crack of separation d, we obtain

$$\frac{Q}{\Delta h} = \left[\frac{W_n - [w_0 + (\epsilon A P_0)^{1/2} \exp((d_0 - d)/2\Lambda)]}{2} \right] \frac{\rho g}{12 \mu L} d^3 \tag{8}$$

Equation (4) fits the closure data (Figure 7) within the experimental error when pore pressure is fixed. Equation (8) shows why the permeability decreases faster with d than d^3 (Figure 5). *Witherspoon et al.* [1980] assumed that w was a constant and introduced a factor f to account for it. We show here (equation (6)) that w is a function of P and hence d.

If A_c is only a function of load and then if a pore pressure P_p is present,

$$A_c = \epsilon PA - \epsilon P_p(A - A_c) \tag{9}$$

and we see why P_p plays a smaller role than P in changing the aperture and permeability. The above result was discussed by *Kranz et al.* [1979].

Acknowledgments. This work was supported by Department of Energy contract DE-AS02-76ER04054. Lamont-Doherty Geological Observatory contribution #3104.

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