Deviatoric Stressitis: A Virus Infecting the Earth Science Community

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This note serves as a warning that a recently mutated virus, deviatoric stressitis, has infected many in the Earth science community. The good news is that this virus, nicknamed deveitis (dee vee' i-tis), is not lethal and its victims can hope for a complete recovery. The bad news is that deveitis has spread like wildfire and leads to great confusion about Earth stress in papers, abstracts, and discussions by any of its victims.

Deveitis is easy to recognize. The most common symptom is the use of the term, deviatoric stress, in papers, abstracts, and discussions.

Examples are numerous. A recent paper[1] from the experimental rock mechanics community stated that ". . . the deviatoric stress was . . ." and then called attention to a figure on the same page. On glancing at the figure, the reader finds a stress-strain plot with the vertical axis labeled, differential stress, $\sigma_d=\sigma_1-\sigma_3$. Obviously, a mild case of deveitis lead to a text error with that noknow term, deviatoric stress, replacing the term, differential stress.

Because the author's intention is clear, we can give the author the benefit of the doubt and assume that a bug in the spell checker of his/her word processor caused an exchange of terms during final manuscript preparation. This example illustrates a troublesome aspect of deveitis; it is difficult to tell whether the disease is a computer bug affecting word processing applications or whether the victim is the host of a virus. What is deviatoric stress? A novitiate, yet to contract the virus, will go to the literature [e.g., *Jaeger and Cook*, 1969; *Means*, 1976; *Suppe*, 1985; *Engelder*, 1993] for a definition of deviatoric stress. He/she will find that stress at a point, $[\sigma_{ij}]$, is characterized by more than one combination of six independent parameters. Given an arbitrary coordinate system these six parameters are in the matrix:

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \qquad (1)$$

where $\sigma_{12}=\sigma_{21}$, $\sigma_{13}=\sigma_{31}$, $\sigma_{23}=\sigma_{32}$. There exists another coordinate system in which the off-diagonal components (that is, σ_{ij}, $i\neq j$) of the stress matrix go to zero and the diagonal components (that is, σ_{ij}, $i=j$) become principal stresses. In this new coordinate system, the coordinate axes are parallel to the direction of the principal stresses. The principal stresses are the eigenvalues of the matrix in equation (1) and the orientations of the principal stresses are given by the eigenvectors in the old coordinate system so that

$$[\sigma'_{ij}] = \begin{bmatrix} \sigma'_{11} & 0 & 0 \\ 0 & \sigma'_{22} & 0 \\ 0 & 0 & \sigma'_{33} \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \qquad (2)$$

In the case of equation (2), the three additional parameters that specify the stress at a point are the eigenvectors. Although the prime (') is used to indicate components of principal stress, it is understood that $[\sigma'_{ij}] = [\sigma_{ij}]$.

Deviatoric stress at a point, $[\delta\sigma_{ij}]$, is derived by subtracting the mean of the normal stress components of the stress matrix (that is, the diagonal components) from each diagonal component of $[\sigma_{ij}]$. The mean stress, σ_m, is an invariant so that

$$\sigma_m = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3}. \qquad (3)$$

Deviatoric stress is a matrix

$$[\delta\sigma_{ij}] = \begin{bmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{bmatrix}. \qquad (4)$$

If the principal stresses and their orientations are known, as is the usual case in experimental rock mechanics and mineral physics, then

$$[\delta\sigma'_{ij}] = \begin{bmatrix} \sigma'_{11}-\sigma_m & 0 & 0 \\ 0 & \sigma'_{22}-\sigma_m & 0 \\ 0 & 0 & \sigma'_{33}-\sigma_m \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1-\sigma_m & 0 & 0 \\ 0 & \sigma_2-\sigma_m & 0 \\ 0 & 0 & \sigma_3-\sigma_m \end{bmatrix} \qquad (5)$$

where $[\delta\sigma'_{ij}] = [\delta\sigma_{ij}]$.

The confusion caused by deveitis is best illustrated by a recent AGU paper from the mineral physics community. This paper reports the strength of a common mineral during unconfined creep tests on right-circular cylinders subject to axial compression. Such a loading configuration imparts a uniaxial stress state on the sample [*Means*, 1976]. In terms of principal stresses, σ_1 = a positive value, $\sigma_2 = 0$, and $\sigma_3 = 0$.

[1] Specific citations are omitted to protect the reputations of the innocent victims.

[2] Unfortunately, the disease is also rampant in Europe.

The author of the AGU paper in question, a victim of the more advanced stage of deveitis, wrote that the deviatoric stress on the sample was 1 MPa. The Earth science community has every reason to be confused by this statement because it implies that deviatoric stress is a single number which is not the case according to equations (4) and (5). With justification, the novitiate might assume that 1 MPa is the component of deviatoric compression along the cylinder axis, in which case $\delta\sigma'_{11} = 1$ MPa (Figure 1). The mean stress, σ_m, during uniaxial stress tests is one third the axial stress, σ'_{11} $(=\sigma_1)$, on the cylinder. Knowing that σ'_{22} $(=\sigma_2)=\sigma'_{33}$ $(=\sigma_3)=0$ in a uniaxial stress experiment, the novitiate will solve for σ'_{11} $(=\sigma_1) = 1.5$ MPa and $\sigma_m = 0.5$ MPa.

An alert novitiate will then understand that during this creep experiment the cylinder was subject to components of deviatoric **tension**, $\delta\sigma'_{22} = \delta\sigma'_{33} = -0.5$ MPa, each oriented normal to the cylinder axis. Although it might seem that Figure 1 shows two different stress states, they are, in fact, the same. Upon reading this particular AGU paper I was immediately suspicious that the author suffered from deveitis. A positive diagnosis was confirmed when the author told me that the creep experiment in question was loaded with an axial stress σ'_{11} $(=\sigma_1) = 1$ MPa, not 1.5 MPa as implicitly stated in his/her paper. In this experiment, $\sigma_d = \sigma_1$. Not only was the author infected, but the reviewers and the AGU editor also suffered from deveitis to some degree. If not, why would the reviewers have permitted publication of a paper giving the impression that the mantle is 50% stronger than otherwise observed?

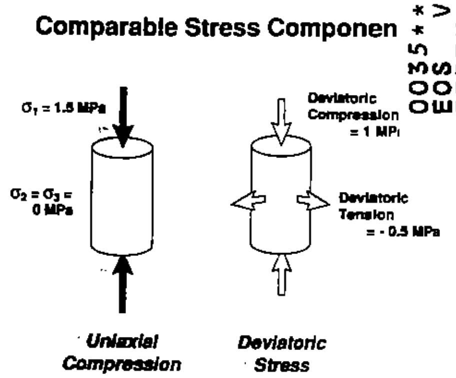

Fig. 1. Two renditions of the same stress state within an unconfined creep test subject to a deviatoric compression of 1 MPa. During this test σ_d=1.5 MPa and σ_1=1.5 MPa.

Deveitis is insidious. At a recent meeting hosted by a reputable U.S. government agency[2], each of the three conveners described various situations where the strength of the crust was an issue. During their presentations, all three suffered mild cases of deveitis. Worse yet, another geophysicist at the same meeting suffered from an acute attack of the virus when stating that the *no-know* stress was responsible for crack closure. Unwittingly, this scientist failed to recognize that by subtracting the mean normal stress he/she was calling upon deviatoric tension to close the crack in question when, in fact, total normal stress does the job. Fortunately for the Earth science community, these victims of deveitis made miraculous recoveries once they became aware of their infection. Full recovery is usually manifested by a feeling of chagrin and a nervous laugh.

On warning my colleagues about deveitis, many fail to give a clear definition for deviatoric stress. The consensus is that the no-know stress has something to do with causing faults to slip and rocks to break or flow, but exact details are fuzzy. Some say the no-know stress is equivalent to shear stress, others think differential stress, tectonic stress, lithostatic stress, axial stress, crack normal stress, sliding stress, stress drop, friction, and the list continues. Some definitions come close but still miss the mark. A widely read graduate text states that horizontal normal stress has two parts, a lithostatic contribution (ρgz where r is density and z is depth) and a tectonic contribution known as deviatoric stress. Oh dear! The problem with this definition is that the mean stress and the overburden stress, ρgz, will be the same only under very unlikely circumstances.

Although difficult to track in the literature, the recent, full blown epidemic of deveitis may be traced back 14 years to the publication of one of the first special issues of *JGR–Solid Earth*, the "Conference on Magnitude of Deviatoric Stresses in the Earth's Crust." As a participant in the "deviatoric stresses" conference, I was in awe of colleagues so nimble that they could describe the Earth's strength in terms of a matrix when I could only handle a single quantity,

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σ_d. Of course, that *JGR* conference volume had and still has high, and infectious, visibility.

When first encountering the no-know term in a paper or discussion, the novitiate is best advised to assume that the author is a victim of deveitis. Usually, the victim of this dread disease will use the no-know stress when he/she actually means either σ_d or maximum shear stress. However, it is equally important for the novitiate to appreciate that some authors correctly use the matrix, deviatoric stress, $[\delta\sigma_{ij}]$. In order to understand the proper use of deviatoric stress, $[\delta\sigma_{ij}]$, one might refer to papers by *Bott and Dean* [1972] or *Etchecopar et al.* [1981] as guides. *Bott and Dean* [1972] describe the stresses found within rifted margins such as the North Sea.

In the case of a young continental margin dominated by listric normal faults, it is correct to say that the component of deviatoric stress normal to the margin is a deviatoric tension. Despite the presence of deviatoric tension, all three principal stresses at depth within rift basins are highly compressive.

Deviatoric stress has a particularly important application in the derivation of stress orientation and magnitude from fault slip data [*Angelier*, 1979]. The stress tensor is the sum of two matrices: $[\delta\sigma'_{ij}]$ and a diagonal matrix with the mean stress as components

$$[\sigma'_{ij}] = \begin{bmatrix} \sigma'_{11} - \sigma_m & 0 & 0 \\ 0 & \sigma'_{22} - \sigma_m & 0 \\ 0 & 0 & \sigma'_{33} - \sigma_m \end{bmatrix} + \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}. \quad (6)$$

Etchecopar et al. [1981] call the latter part of equation (6) the isotropic pressure part of the stress tensor. Isotropic pressure cannot be measured using fault-slip techniques. If

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the isotropic pressure part of the stress tensor is just a unit matrix, **I**, multiplied by a constant, b, and if the deviatoric part is a normalized deviatoric matrix, $\mathbf{D}_o$, multiplied by a constant, a, then

$$\mathbf{S} = a\mathbf{D}_o + b\mathbf{I} \quad (7)$$

where $\mathbf{S} = [\sigma'_{ij}]$. One common method for normalizing the deviatoric matrix is to set σ_1-$\sigma_3 = 1$. With normalization, the principal components of $[\delta\sigma'_{ij}]$ vary from zero to one. Thus given the orientation of the principal stresses, **S** can be represented by three parameters other than the principal stresses. These are the mean stress (the isotropic pressure), the differential stress, and a stress ratio which is defined as

$$R = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \quad (8)$$

Fault-slip data allow the determination of four parameters, the orientation of each of the three principal stresses and R.

In summary, deveitis is not fatal but can lead to unending confusion about Earth stress. Use of the term. deviatoric stress should be completely eliminated from the literature except for special applications such as fault slip analysis.

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